

Semester Two Examination, 2021

Question/Answer booklet

MATHEMATICS
SPECIALIST
UNITS 1&2

**SOLUTIONS**

Section Two:
Calculator-assumed

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| WA student number: In figures |  |  |  |  |  |  |  |  |  |  |

 In words

 Your name

|  |  |
| --- | --- |
| Number of additionalanswer booklets used(if applicable): |  |

## Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number ofquestionsavailable | Number ofquestions tobe answered | Workingtime(minutes) | Marksavailable | Percentageofexamination |
| Section One:Calculator-free | 8 | 8 | 50 | 50 | 35 |
| Section Two:Calculator-assumed | 13 | 13 | 100 | 92 | 65 |
|  |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.
Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (92 Marks)

This section has**thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 (6 marks)

(a) Determine the vector projection of on . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates appropriate methodü correct vector |

(b) Determine the value(s) of so that the vectors and are

(i) parallel. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates equation using of ratio of coefficientsü correct value |

(ii) perpendicular. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates equation using scalar productü correct values |

Question 10 (7 marks)

(a) Five-digit odd numbers are to be made using the digits and . Determine how many such numbers exist if the number must exceed and no digit may be used more than once in a number. (3 marks)

|  |
| --- |
| Solution |
| End with or : End with or : Total possible numbers:  |
| Specific behaviours |
| ✓ splits into mutually exclusive casesü correctly counts at least one caseü calculates total |

(b) The library in a small guesthouse has different books, of which are non-fiction and the remainder fiction. Determine the number of different ways that a guest can select four books if they want

(i) the same number of fiction and non-fiction books. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates correct methodü correct number of ways |

(ii) more fiction than non-fiction books. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates correct methodü correct number of ways |

Question 11 (7 marks)

Two transformation matrices are and .

Triangle has an area of cm2, with vertices at and .

(a) Determine the coordinates of after the triangle has been transformed by matrix .

 (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates pre-multiplication by ü correct matrix productü correctly lists set of coordinates |

(b) Use the geometric transformation to explain why the determinant of is . (1 mark)

|  |
| --- |
| Solution |
|  represents a reflection, the area of triangle does not change and so determinant is . |
| Specific behaviours |
| ✓ reflection will not change area |

(c) Use the geometric transformation to explain why , where is the identity matrix. (1 mark)

|  |
| --- |
| Solution |
|  represents two reflections in the same line, and so the triangle will be back where it started, with the same coordinates. |
| Specific behaviours |
| ✓ two reflections in same line |

(d) Determine the area of after the triangle has been transformed by matrix . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ calculates determinantü calculates new area |

Question 12 (8 marks)

(a) Write the converse of the true statement 'if a figure is a square then it has four congruent sides' and use an example or counter-example to briefly discuss the truth of the converse.

 (2 marks)

|  |
| --- |
| Solution |
| Converse: If a figure has four congruent sides, then it is a square.The converse is false, as the figure could be a rhombus. |
| Specific behaviours |
| ✓ correct converseü states false with counter-example |



(b) Points and lie as shown on a circle
with centre so that is a diameter,
 and .

 Determine the size of

(i) . (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ ✓ ✓  |

(ii) . (1 mark)

(iii) . (1 mark)

(c) Two chords of a circle, and , intersect at so that cm, cm and cm. Determine all possible lengths of . (3 marks)

|  |
| --- |
| Solution |
| Using intersecting chord theorem, .Let , so that Hence cm or cm. |
| Specific behaviours |
| ✓ identifies required lengthsü forms quadratic equationü states both values |

Question 13 (8 marks)

In triangle , is the midpoint of and is the midpoint of . Let and .

(a) Show that . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates logical stepsü uses correct vector notation throughout |

The position vector of is , position vector of is and is the origin.

(b) Determine a unit vector in the same direction as . (2 marks)

|  |
| --- |
| Solution |
| *\*NB might use CAS for last step* |
| Specific behaviours |
| ✓ calculates ü states unit vector |

(c) Show that is perpendicular to . (2 marks)

|  |
| --- |
| Solution |
| Hence vectors are perpendicular. |
| Specific behaviours |
| ✓ calculates ü shows scalar product is zero |

(d) Determine the size of . (2 marks)

|  |
| --- |
| Solution |
| *\*NB might use CAS* |
| Specific behaviours |
| ✓ indicates suitable methodü correct angle |

Question 14 (7 marks)

(a) In trapezium , and are diagonals, and is parallel to . Use a vector method to prove that . (3 marks)

|  |
| --- |
| Solution |
| Note that and . |
| Specific behaviours |
| ✓ expression for ü expression for ü adds expressions and simplifies |

(b) In rectangle , let and . Use a vector method to prove that if the diagonals and are perpendicular, then is a square. (4 marks)

|  |
| --- |
| Solution |
| The diagonals are the vectors and **.**If diagonals are perpendicular, then and so:Hence must be a square since it is a rectangle with equal length sides. |
| Specific behaviours |
| ✓ determines vectors for diagonals✓ uses scalar product✓ expands and simplifies scalar product✓ explains that sides are equal length |

Question 15 (6 marks)

Consider the following statement:

For two integers if is a multiple of then at least one of is even.

(a) Write the contrapositive of the statement. (1 mark)

|  |
| --- |
| Solution |
| For two integers if both are odd then is not a multiple of . |
| Specific behaviours |
| ✓ correct contrapositive |

(b) Prove that the statement is true. (5 marks)

|  |
| --- |
| Solution |
| Proof of contrapositive:If both odd, then and , where both integers.HenceHence will never be a multiple of as it is always one more than a multiple of .Since the contrapositive statement has been proved to be true then it follows that the original statement must also be true. |
| Specific behaviours |
| ✓ attempts to prove contrapositive and states truth of contrapositive implies truth of original statementü uses form form for odd numbers ü substitutes for and expandsü factors out ü explains why contrapositive true |

Question 16 (8 marks)

The height of the tide, cm, of the sea above the mean level at time hours after midnight one day is given by

(a) Express in the form , where and . (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ value of ü value of ü correct expression for  |

(b) Determine, to the nearest minute, the time of the first high tide. (2 marks)

|  |
| --- |
| Solution |
| Require . Hence at or am. |
| Specific behaviours |
| ✓ time in hoursü time of day, to nearest minute |

(c) Sketch the graph of the height of the tide on the axes below. (3 marks)

|  |
| --- |
| Solution |
| See graph |
| Specific behaviours |
| ✓ vertical scale and interceptü rootsü smooth sinusoidal curve |



Question 17 (7 marks)

Three forces and act on a small body, where N, N and
 N.

(a) Determine the magnitude of the resultant force and the angle between the resultant force and the vector . (3 marks)

|  |
| --- |
| Solution |
| Hence resultant has a magnitude of N and makes an angle of with . |
| Specific behaviours |
| ✓ correct sum in component formü calculates magnitudeü calculates angle |

(b) Two of the forces, and , can be multiplied by scalars and respectively so that the three forces are in equilibrium. Determine the value of and the value of . (4 marks)

|  |
| --- |
| Solution |
| Resolving in and directions:Solving simultaneously gives |
| Specific behaviours |
| ✓ writes vector equation equal to ü forms two equationsü value of ü value of  |

Question 18 (7 marks)

(a) people are asked to choose two different letters from those in the word GAMBLER and write them down in order. Use the pigeonhole principle to prove that at least three people will write the same pair of letters in the same order. (3 marks)

|  |
| --- |
| Solution |
| There are different ordered pairs of letters, each of which is a pigeonhole.Using the pigeonhole principle, if pigeons (the number of pairs of letters written by people) are placed into pigeonholes, then at least one pigeonhole will contain or more pigeons.Hence at least people will write the same pair of letters in the same order. |
| Specific behaviours |
| ✓ obtains number of permutationsü indicates pigeons and pigeonholesü uses pigeonhole principle to complete proof |

(b) Three character codes, such as TCU, are made using three different letters chosen from the word DISCOUNT. Determine the proportion of all possible codes that start with a D or end with a T. (4 marks)

|  |
| --- |
| Solution |
| Start with D: codes.End with T: codes.Start with D and end with T: codes.Start with D or end with T: codes.There are different codes.Hence required proportion is . |
| Specific behaviours |
| ü and ü ü ✓ number of possible codes and writes proportion |

Question 19 (7 marks)

Airport B lies km due east of airport A, and in the region of the airports a wind of km/h is blowing from the northeast.

A small plane, with a cruising speed of km/h, leaves airport A. The pilot, not aware of the wind and intending to fly to airport B, steered the plane on a bearing of .

Assuming that the pilot does not realise their mistake, determine how close the plane will come to airport B if it continues to fly for several hours on the same bearing.

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ appropriate sketch/explanation for and ü equation for ü solves for ü equation for ü solves for ✓ appropriate sketch/explanation for closest distanceü calculates closest distance |

Question 20 (7 marks)

Triangle has vertices and .

 is rotated clockwise about the origin to form triangle .

(a) Determine the coordinates of . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ rotation matrixü correct coordinates |

 is reflected in the line to form triangle .

(b) Determine the coordinates of . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ reflection matrixü correct coordinates |

(c) Determine matrix that will transform to . (3 marks)

|  |
| --- |
| Solution |
| Matrix for is inverse of that used in (a).Matrix for is same as used in (b).Hence |
| Specific behaviours |
| ✓ matrix for ü indicates correct order of multiplicationü correct matrix  |

Question 21 (7 marks)

The diagram, not drawn to scale, shows
vertices and of an isosceles triangle
lying on a circle so that .

Lines and are tangential to the
circle at and respectively.

 is a cyclic quadrilateral.

Let .

(a) Determine, with reasons, the size of in terms of . (5 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ expression for with reason✓ expression for with reason✓ expression for with reason✓ expression for with reason✓ expression for with reason |

(b) Hence determine the range of values for the size of in degrees. (2 marks)

|  |
| --- |
| Solution |
| For figure to exist, .Hence Range is . |
| Specific behaviours |
| ✓ indicates correct domain for ü correct range, including inequalities |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

© 2021 WA Exam Papers. Kennedy Baptist College has a non-exclusive licence to copy and communicate this document for non-commercial, educational use within the school. No other copying, communication or use is permitted without the express written permission of WA Exam Papers. SN245-181-4.